PROBLEMS OF THE
56th—28th INTERNATIONAL—RUDOLF ORTVAY
PROBLEM SOLVING CONTEST IN PHYSICS
30 January—9 February 2026

1. "The North Building in the Lágymányos Campus of Eötvös University features a nearly hemispherical hall, which is quite charateristic from its echoing."— this is how, in the (binary) 110000th Ortvay competition the text of the 10th problem began. So it is natural that now, in the 111000th competition, we revisit the topic.


If we speak near the middle of the hall, we hear the echo surprisingly strongly. And if we clap, we hear periodically repeating, fading echoes. Naively, one would think that the repetition period is identical to the time it takes for sound to make the round trip between the floor and the ceiling. However, based on the hall's height of $h \approx 10.8$ m and the sound speed of 344.7 m/s, we get approximately 0.063 s for this, which is clearly too frequent a repetition. Even without measurement, it can be determined that the number of repetitions is around 4—5 per second.

Why is this so, and exactly what value do we get for the period time?

For the calculation, you need to know that the ceiling of the hall is a spherical surface with radius $R \approx 15.2$ m, meaning that even if the spherical surface were continued down to the floor, it would not form a complete hemisphere. The base of the hall is, of course, flat. Modeling the sound waves with perfectly elastically bouncing particles, let us first examine, for a given value of h, what is the optimal value of R from the perspective of the first echo's strength, for sound waves traveling near the vertical axis of symmetry! Does our height significantly influence this and the repetition period? How much do these results change if we don't only consider particles traveling near the axis of symmetry!

(Zoltán Kaufmann)

2. We would like to measure the radius of the Earth. The 'mass' of a ceramic mug was measured to be 469.90 grams on the 3rd floor of the North Building of ELTE's Lágymányos campus in Budapest, and as 469.86 grams on the top of János Hill in Budapest using an exceptionally high resolution digital scale. Let us consider the shape of János Hill as a right cone with a half-opening angle of 45 degrees. What is the radius of the Earth and its experimental uncertainty?

(Gábor Veres and Imre Derényi)

3. We measured the temperature in Balatonfőkajár, Újpest and Dunaújváros on a summer day, from noon to 10 p.m., every 10 minutes, in degrees Celsius. The measured data are provided below. As an approximation, let us assume that a completely straight, very long, cold front moving at a constant speed and in a constant direction passed through the country. How much later did the front arrive at Kecskemét than at Balatonfőkajár? Let us also give the experimental uncertainty of this quantity.

   Újpest (coordinates: 47.5670, 19.0980):

   28.5, 28.5, 28.4, 28.7, 28.9, 28.8, 28.2, 28.3, 28.1, 28.2, 28.0, 27.9, 27.8, 27.9, 28.1, 27.9, 27.9, 27.9, 27.8, 27.7, 27.6, 27.1, 26.4, 25.7, 24.7, 23.8, 22.8, 21.9, 21.1, 19.8, 18.2, 17.7, 17.4, 17.3, 17.1, 16.8, 16.6, 16.4, 16.3, 16.2, 16.2, 16.2, 16.4, 16.5, 16.6, 16.4, 16.3, 16.1, 16.1, 16.0, 16.0, 15.9, 15.9, 15.7, 15.5, 15.4, 15.3, 15.3, 15.3, 15.4, 15.5.

   Dunaújváros (coordinates: 46.9506, 18.9522):

   27.0, 27.7, 28.3, 28.2, 28.9, 29.0, 28.9, 28.9, 30.0, 29.6, 29.8, 30.5, 30.2, 30.9, 30.0, 30.0, 31.2, 30.9, 31.1, 29.6, 29.5, 30.1, 26.8, 24.8, 24.0, 23.5, 22.4, 22.6, 21.1, 20.2, 20.5, 20.3, 19.9, 19.6, 19.4, 19.1, 18.7, 18.4, 18.3, 18.3, 18.0, 18.2, 18.0, 18.3, 18.0, 17.7, 17.5, 17.2, 17.1, 17.0, 16.9, 16.8, 17.1, 17.2, 17.2, 17.3, 17.0, 16.9, 16.8, 16.7, 16.7.

   Balatonfőkajár (coordinates: 47.0194, 18.2158):

   28.9, 29.0, 29.1, 29.1, 29.5, 29.9, 30.1, 30.2, 29.7, 30.5, 30.5, 30.6, 30.7, 30.8, 30.9, 30.9, 29.9, 26.9, 25.5, 24.9, 23.2, 21.9, 20.8, 20.4, 20.4, 19.8, 19.6, 19.3, 19.2, 18.4, 18.3, 17.6, 17.0, 16.7, 16.1, 16.1, 16.1, 16.1, 16.1, 16.1, 17.4, 17.4, 17.4, 17.5, 17.5, 17.5, 17.3, 17.3, 17.3, 17.3, 17.3, 17.3, 17.2, 17.0, 16.8, 16.5, 16.4, 16.4, 16.6, 16.7, 16.7.

   (Gábor Veres)

4. A rigid hemisphere lies on a horizontal plane on its flat surface. The hemisphere is able to slide on the plane without friction. An other small body may move on the surface of the hemisphere without friction. The air drag is negligible. Initially, the small body is located at the top of the hemisphere, the system is at rest. Then the hemisphere is jolted horizontally with a very short duration push.

   When the small body reaches the horizontal base plane, it leaves a mark at the point of contact.

   a) What is the infimum of the distance between the point of contact and the edge of the hemisphere? Provide a numerical result, expressed in units of the radius of the hemisphere. Under what physical conditions can this infimum be approached?

   b) Study possible scenarios for the process of this phenomenon.

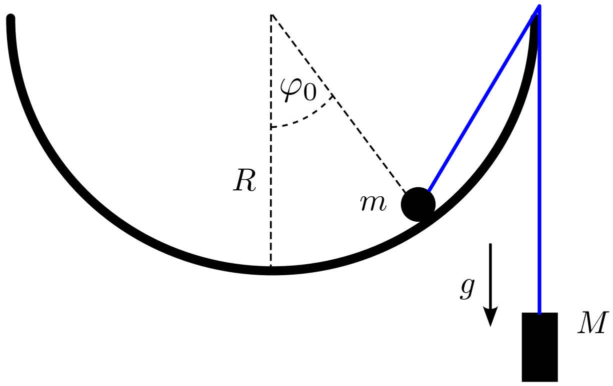
   (Gyula Dávid)

5. A cylinder containing compressed gas is floating in space, completely weightless. A micrometeor punctures a tiny hole in the wall of the cylinder (but slows down just enough so it does not escape on the other side). Due to the small mass of the meteor, the collision does not change either the velocity or the state of rotation of the cylinder; after the collision, the angular velocity and the velocity of the center of mass of the cylinder are both zero.

   The gas starts to escape from the cylinder through the hole at a constant velocity, which causes the cylinder to move. Let us consider the cylinder (with its contents) as a rigid body; during the motion, the total mass of the cylinder and the pressure inside can be assumed approximately constant, and the motion is non-relativistic. We find that after a while the velocity of the center of mass of the gas cylinder becomes constant, although the outflow of gas is continuous. What is the condition for this? Let us express the magnitude of the gradually achieved constant velocity in terms of the relevant parameters!

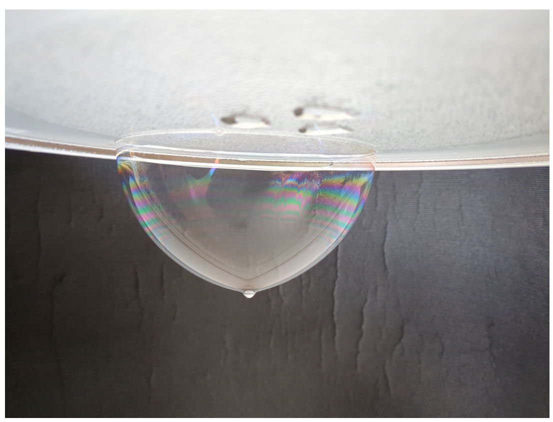
   (Máté Vigh)

6. In a cylindrical fixed trough with radius R, a point mass m is held on a rope as shown in the figure, with the rope passing over the trough and the other end attached to a mass M hanging vertically. The acceleration due to gravity is g. Initially, the mass m is at rest at an angle φ_0 ($-\pi/2 \le \varphi_0 \le \pi/2$). Study the motion of the two bodies as a function of the given parameters.



(József Cserti)

7. What is the shape of a bubble hanging from a horizontal ceiling in a uniform gravitational field? Assume that the surface mass density of the bubble's film is uniform. What is the pressure inside the bubble? Perform numerical calculations if it is no longer possible to proceed analytically, and attach the resulting figures to the solution. There is no need to submit code, only to summarize in writing which methods were required.



(Máté Tibor Veszeli)

8. The inhabitants of Flat Earth live on two square sides of a rectangular prism with side length a, thickness h, and homogeneous density ϱ. The government of Flat Earth has had enough of the division. The Prime Minister issued an order: in the middle of both sides of Flat Earth, they should start drilling a small hole with a cross-section A ($A \ll a^2$). Trusting that the two holes will eventually meet, they will succeed in uniting the inhabitants of both sides of Flat Earth. The energy required to transport the rock to the surface during excavation would be provided by the previously ordered nuclear power plant with capacity P. Money is no object. If everything goes well, how long will the excavation take? The excavated rock is currently near the hole, and its spreading is part of a later project.

(*Data*: $a = 10000\,\mathrm{km}$, $h = 2000\,\mathrm{km}$, $A = 5\ \mathrm{m}^2$, $\varrho = 5500\ \mathrm{kg/m}^3$, $P = 1000\,\mathrm{MW}$)

(József Cserti)

9. A toilet paper roll rests on a toilet paper holder with a horizontal axis, and a sheet hangs down from its end. With what speed must one pull suddenly the end of the hanging part so that the last sheet tears off? By introducing relevant parameters, construct a physical model to describe the phenomenon.

(Dávid Szepessy)

10. Two bodies of mass m can move along a straight line, and the interaction is given by a pair potential $V(x_1 - x_2)$. Let us assume the symmetry $V(x) = V(-x)$. The potential is unknown, and we intend to obtain information about it, using a thought experiment which is motivated by particle physics. We initialize the two particles far apart, with velocities v and $-v$. We don't have an apparatus to follow the entire motion, but we can measure the trajectories of the two particles after a long time. Let us assume that the two masses don't stick together, so the two particles either pass each other or there is a full reflection. If there was no interaction, then the trajectory of one of the particles could be described in a suitable coordinate system with the equation $x = vt$. In the case of the interaction let us assume that a long time before scattering, the trajectory is $x = vt$ to a good approximation. After scattering we find $x = vt + d$ to a good approximation. What can the sign of the shift d tell us about whether the potential is attractive or repulsive? Given the velocity dependence of the displacement d, how can we deduce the shape of the potential? Let us examine some simple cases!

(Balázs Pozsgay)

11. A cylinder with homogeneous mass distribution, density ϱ, radius a and length h is cut into two equal halves in a plane perpendicular to its symmetry axis. What is the attractive force between the two halves if the distance between them is negligible compared to the dimensions of the cylinder? In the absence of an analytical result, calculate the attractive force numerically for different values of h/a. Determine the approximate analytical form of the attractive force in the case of $h \gg a$ and compare it with the numerical results.

(József Cserti)

12. Put the images in chronological order.


Explain what happened based on your physical intuition. Create a physical model that is as accurate as possible, with realistic physical parameters, that can reconstruct the phenomenon. Solve the model analytically or numerically.

(István Csabai)

13. An ellipsoid-shaped rigid body floats on water. The body has uniform density ϱ, while the water density is $\varrho_0 > \varrho$. The semi-axes of the ellipsoid are $a \geq b \geq c$, and we define the ratios $k = b/a$, $l = c/b$, $r = \varrho/\varrho_0$. Depending on the values of $(k,\, l,\, r)$, the body can float in different equilibrium orientations. These equilibria may be stable or unstable and can include inclined configurations characterized by orientation angles (φ, ϑ). Determine the stable and unstable equilibrium orientations of the ellipsoid as functions of $(k,\, l,\, r)$.

(Szilveszter Fehér and Róbert Németh)

14. 'Planet Flatwater' is a solid cylinder of homogeneous rock with a square cross-section. Its average density and total mass are the same as that of Earth. The cylinder rotates once every three hours on its axis. The disk of the planet is covered by a sea, all the way to the Rim, just barely not overflowing.

An intelligent fish species is living in the sea of the planet. They learned about the existence of other Earths (flat and non-flat) and even the very concept of „land" from a radio broadcast from distant Earth, which was broadcasting a scientific debate. They liked the idea very much, and a brave group decided to follow in the footsteps of the amphibians who once conquered the distant Earth—later to grow legs and crawl out onto land. This plan was greatly hindered by the fact that the sea reaches the Rim, so there is no land at all on the disc. (Even the revolutionary fish did not dare to think of crossing the Rim.) Some suggested that one could throw some surface water into space, but this was quickly voted down by the majority of fish who wanted to continue swimming in the sea ("We are the sea!" - they gaped).

Finally, they agreed upon a compromise: they would ask for the help of the Vogons, the popular planetary technicians they had also met from terrestrial radio broadcasts, and have them slow down the rotation of their planet—to the point of stopping it completely. Prostetnic Vogon Jeltz accepted the assignment. The impossibly huge yellow somethings set to work, the reformer fish were soon happy to see the sea recede, and the long-awaited land appeared along the ring of the Rim. It kept growing until the rotation of the planet stopped completely. (The rotation was slowed down adiabatically, so the location of the ocean was always able to adapt to the current angular velocity of rotation without major disturbances.)

a) The task of the participants in the Ortvay competition is simple: calculate and plot the proportion of land area and the greatest depth of the sea during the deceleration process as a function of the planet's rotation angular velocity. Give precise numerical values, with justified calculations to sufficient details. (Assume that the mass of seawater is negligible in comparison to the mass of the rock.)

The joy of the fish preparing to land, however, was soon turned to sorrow, for the land ring was overrun with Vogon fishing tourists, who particularly enjoyed catching the fish swimming around the coast, enthusiastic to become amphibians. When they mentioned this, Prostetnic Vogon Jeltz shrugged his part of the body one would call shoulders: "Don't you wish for dry land? Now we can help you get there."

The fish finally decided to cancel the contract and ask for the planet's rotation to be restored again. Prostetnic Vogon Jeltz nodded after the usual fuss, and the automated yellow somethings began to accelerate the rotation of the planet. But they didn't stop even when the sea reached the Rim again. The Vogons left the impossibly huge yellow somethings behind, and the fish had no access to the construction fleet control system. Then one fine day...

b) The fragmented account of the unfolding evens available in the Galactic Library cuts off here. However, the participants of the Ortvay competition can continue their previous calculations and reconstruct subsequent events, supporting the improbable story with precise numerical values.

(Gyula Dávid)

15. The Hamiltonian of a mechanical system with one degree of freedom is the following:

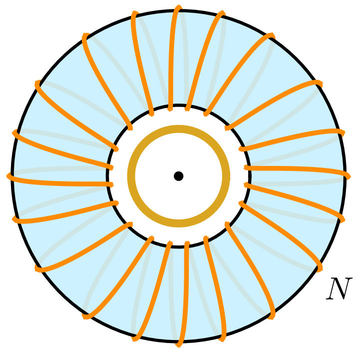
$$H(q,p) \,=\, A\,\,\frac{\tan\,(\alpha\,p^2\,+\,\beta\,q^2)}{(\alpha\,p^2\,+\,\beta\,q^2)^{12}}\,,$$

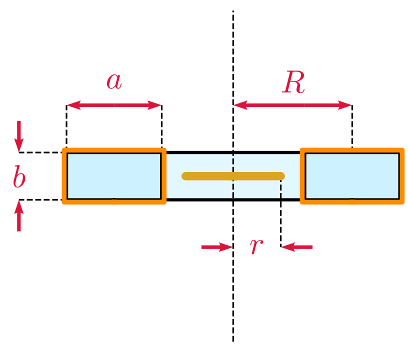
where A, α, and β are real parameters.

Derive the canonical equations of motion and solve them analytically. Sketch the phase space of the system, plot the trajectories. Find the fixpoints and study their stability. Discuss the solutions according to the values of the parameters.

(Gyula Dávid)

16. On a plastic torus of major radius R and a rectangular cross-section of side lengths a and b, we coil N turns of a copper wire, realizing a toroidal inductor ($b \ll R, a$). A circular conductor of radius r is placed concentrically and coaxially with the coil, as seen in the top-view and side-view figures below ($r < R - a/2$, $b \ll R - a/2 - r$).

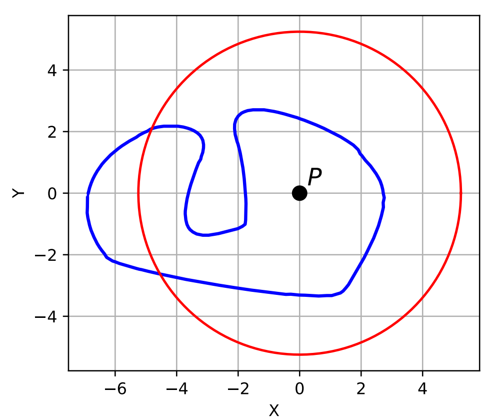




Calculate the mutual inductance of the system. How does this quantity depend on the geometric parameters R, a, and r, and on the exact shape of the copper wire?

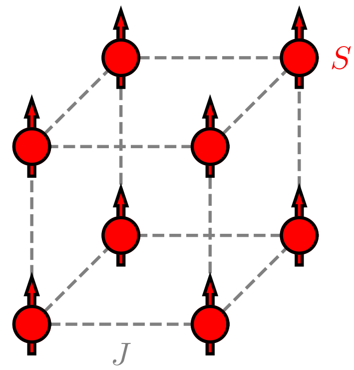
(Róbert Németh)

17. A current of magnitude I flows in the circular conductor wire shown in the figure (red curve). By how much does the magnitude of the magnetic field B change at the center of the circle (at point P) if the unstretchable wire is deformed as shown in the figure (blue curve) while remaining in the plane of the original circle? We expect a numerical result accurate to three decimal places, with a detailed description of the calculation procedure.



(József Cserti)

18. The atoms of a strange molecule are arranged in the vertices of a rigid cube, according to the figure. Each atom possesses an internal angular momentum of magnitude S, and of continuously varying direction (classical spin).



Between the nearest neighbors, there is a Heisenberg-type interaction, with a uniform ferromagnetic or antiferromagnetic exchange constant J. Determine the possible equilibrium configurations of the system and analyze their stability.

(Róbert Németh)

19. An infinite mass object is fixed at the origin of an inertial system. The object generates a four-scalar field $\Phi(r) = G/r$ where G is the scalar charge of the central object and r is the (three-)distance from the origin.

A paralel beam of particles with mass m, scalar charge g and (three-)velocity u arrives from infinity.

Calculate and plot the differential cross section of the scattering process. Discuss the different outcomes.

(Gyula Dávid)

20. The dynamics of an anisotropic two-dimensional electron gas is described by two effective mass parameters, M_x and M_y. The one-particle Hamiltonian of the system is

$$\hat{H} = \frac{\hat{p}_x^2}{2M_x} + \frac{\hat{p}_y^2}{2M_y},$$

where $\hat{p}_x$ and $\hat{p}_y$ are components of the momentum operator. Our goal is to determine these mass parameters in a thought experiment. To this end, we apply a homogeneous magnetic field of flux density $\boldsymbol{B}$ perpendicularly to the system. Following this, there is only a single operation we can perform, but we can repeat it an arbitrary number of times and at arbitrary intervals: the measurement of the angular momentum component $\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$. Is it possible to determine M_x and M_y in this manner? If so, give an example of a suitable measurement protocol. If not, justify why not.

(Róbert Németh)

21. Let us examine the phase transition of a two-dimensional classical interacting gas in a simulation. For the simulation, we can use the following GitHub repository:

https://github.com/veszeli/hamiltonian_gas_dynamics_and_statistics.

The `M2Interaction` class in the `gas_interaction.py` file describes an interaction potential that includes an attractive term, making it suitable for modeling a condensed phase system as well.

$$U_{\text{pair}}(\mathbf{r}_i, \mathbf{r}_j) = \begin{cases} u_{\text{max}} + k_{\text{down}}\, d & \mid d < d_1 \\ u_{\text{min}} + k_{\text{up}}\,(d - d_1) & \mid d_1 < d < d_2 \\ 0 & \mid d_2 < d \end{cases}$$

with $d = \|\mathbf{r}_i - \mathbf{r}_j\|$, $k_{\text{down}} = \frac{u_{\text{min}} - u_{\text{max}}}{d_1}$ and $k_{\text{up}} = -\frac{u_{\text{min}}}{d_2 - d_1}$

Choose the system parameters so that a phase transition is possible, and modify the simulation so that this event can be observed.
At the given parameters, at what temperature will this occur? How would you construct a phase diagram for this system?

Treat the task as an experiment for which a report must be written. No code needs to be submitted; just describe the principles on which you modified or supplemented the code. Prepare informative figures for the main results.

(Máté Tibor Veszeli)

22. Study the expansion of the flat (i.e. the space is not curved, $k = 0$), homogeneous and isotropic universe with Robertson–Walker metric, assuming different models of matter. The cosmological constant Λ may be positive, negative or zero.

a) Let the equation of state be the following: $p = \alpha\varepsilon^2$ where α is a real (positive or negative) coefficient. Derive the relation between the energy density ε and the scale factor $a(t)$.

Derive the Friedmann equations of the expansion and study qualitatively the different storyboards of the expansion. Assuming $\Lambda = 0$ calculate analytically and plot the funcions $\varepsilon(t)$ and $a(t)$.

b) Repeat the above considerations using the equation of state $p = \beta\sqrt{\varepsilon}$ where β is a real (positive or negative) coefficient.

(Gyula Dávid)

23. Consider the single-site (or zero-hopping) limit of the Hubbard model, i.e., when the electrons in the lattice cannot jump to neighboring sites, in the path integral formalism. The Hamiltonian of the full system is simply the sum of the independent, single-site Hamiltonians. One such Hamiltonian is

$$H = U n_\downarrow n_\uparrow - \mu(n_\downarrow + n_\uparrow) \ ,$$

with $U > 0$, and with spin $\sigma = \downarrow$ or $\uparrow$ number operator $n_\sigma = c_\sigma^\dagger c_\sigma$. The electron creation and annihilation operators are respectively c_σ and $c_\sigma^\dagger$. The fermionic coherent state for spin $\downarrow$ / $\uparrow$ electrons described by Grassmann numbers $\psi = (\psi_\downarrow, \psi_\uparrow)$ is

$$|\psi\rangle \equiv |\psi_\downarrow, \psi_\uparrow\rangle = e^{-\sum_\sigma \psi_\sigma c_\sigma^\dagger} |0, 0\rangle \ ,$$

with vacuum state $|0, 0\rangle \equiv |0\rangle$.

Consider the matrix element corresponding to some small imaginary time step $\Delta\tau$

$$e^{-\psi^* \psi} \langle \psi | e^{-\Delta\tau H} | \psi' \rangle \ .$$

Expand the matrix element in $\Delta\tau$. Collect for each order what terms appear and what terms vanish due to the anticommuting nature of Grassmann numbers. Insert the identity

$$1 = \sqrt{\frac{\Delta\tau}{2\pi U}} \int_{-\infty}^{\infty} \mathrm{d}\varphi \ \exp\left[-\frac{\Delta\tau}{2U} \left(\varphi - iU(\psi^* e^{\Delta\tau\mu} \psi') \right)^2 \right]$$

into the matrix element and cancel out one of the terms that is quartic in the fermion fields. Consequently, the dynamical auxiliary field φ will appear, alongside with the coupling $\propto \varphi\psi^*\psi'$. Are there any more fourth-order terms left? If so, justify why they can be neglected still if $\Delta\tau \to 0$.

Show that the matrix element can be recast into the form

$$\sqrt{\frac{\Delta\tau}{2\pi U}} \int_{-\infty}^{\infty} \mathrm{d}\varphi \ e^{-\Delta\tau\varphi^2/(2U)} \exp\left[\psi^* e^{\Delta\tau(\mu + i\varphi + U/2) + \mathcal{O}(\Delta\tau^{3/2})} \psi' \right] \ .$$

Then, dropping the $\mathcal{O}(\Delta\tau^{3/2})$ term, write down the single-site partition function as a path integral over fields φ, ψ, ψ^*. For now keep $\Delta\tau$ finite, and write it as $\Delta\tau = \beta/N$. For finite N, carry out the integral over the fermionic fields exactly.

At this point the finite-N effective action $S_{\text{eff}}^N[\{\varphi\}_{k=1}^N]$ depends only on the auxiliary field φ. Take the $N \to \infty$ limit, then write the auxiliary field $\varphi(\tau)$ as a Matsubara sum. Determine which Matsubara modes contribute from the fermionic part of the action, and which modes give only a Gaussian (free) term. At this point the partition function is reduced to a one-dimensional integral. Evaluate this integral exactly.

(Dávid Pesznyák)

24. Dr. Absoluto Zero, the Hereditary President of the small but rightly forgotten equatorial country of Gummy Coast (Father of the Fatherland, Protector of Orphans and Widows, Guardian of the Rubber Palms, etc.), sent his scouts to the neighboring empires. Their task was to gather information about the research they conduct that might have an impact on Gummy Coast's national security. Based on the reports, it turned out that for some time now, they had been trying to achieve Quantum Supremacy everywhere, although opinions differed as to whether this Quantum Supremacy, if achieved, would be truly practical. After serious consideration, the dictator summoned his Chief Court Physicist and told to him:

"We cannot allow our beloved Republic to fall behind the neighboring empires! It is very important that we also urgently achieve Quantum Supremacy. But we must not achieve some useless Quantum Supremacy! Our Quantum Supremacy must be useful and hit hard! You know what? Specifically hit hard! (Here he looked up sideways.) Let's create the Quantum Cannonball of Gummy Coast!"

Dr. Ali Tudde Mynek nodded frowning. Some obvious technical difficulties, such as decoherence and the public procurement administrative difficulties flashed through his mind, and he was trying to push them away when the President—excited by his own genie—continued:

"And the Quantum Cannonball should be faster! Reach its target faster than a regular cannonball! That would be a truly useful Superiority!" he said, and His Absoluteness stormed off.

The physicist walked back to the Rubber Physics Research Institute with concern, and that same day he called an old acquaintance who worked at IBM (International Ballistics of Matter). After a long conversation, they agreed that it might be worth a feasibility study and a prototype development tender. This would also buy them time, and then... They specified the goals to be achieved in the tender to be announced:

"Quantum Cannonball, marching towards Quantum Supremacy! Prepare a one-dimensional single-particle state (denote the position variable by x) whose support is the interval $[0, a]$. Denote the expectation value of the particle's momentum by p. Calculate how long it would take a reference (same mass, same momentum, also performing one-dimensional motion) classical cannonball to reach the target located at $10\,a$, and denote this time by T. To avoid any questions about whether the Quantum Cannonball is really faster than the reference cannonball, when calculating T, let us start the classical cannonball from the point with coordinate a. The goal for the Quantum Cannonball is to reach the target sooner than T. That is, the probability that the Quantum Cannonball launched at time 0 will be located to the right of the target point ($x > 10\,a$) after exactly T time should be maximal."

Ali Tudde Mynek then considered why he should do the calculations himself. Therefore, he forwarded the call for applications to the participants of the Ortvay competition, asking them to put a feasibility study on the table.

The prototypes will be presented at Round Square next to the University of Science and Technology, named after Dr. Absoluto Zero, in the presence of the dictator sometime during the year. Due to urgency, decoherence will not be addressed in the preliminary solution of the task. However, we request the studies to be submitted within 10 days. The study should only include the shape of the wave function of the Quantum Cannonball and how the applicant arrived at its result. Given the high national security importance of the task, we accept theoretical considerations, approximations and numerical calculations as well.

(Győző Egri)

(Győző Egri)

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