THE 45th—17th INTERNATIONAL—RUDOLF ORTVAY PROBLEM SOLVING CONTEST IN PHYSICS
22 October—3 November 2014

1. A group of people sitting symmetrically at a round table play the following game. They pass a small ball, always to the immediate neighbour, with equal probability 1/2 to the left or to the right. The game always starts at same person, the 'master', and one turn of the game finishes in the case when everybody has received the ball at least once. The winner is the last one taking the ball (evidently the master can never win). If the turn is over with the winner announced, the master starts a new turn.

   Determine the winning probabilities around the table. Give a suggestion for a new player, which additional seating position offers the highest chance of winning.

   (Győző Egri)

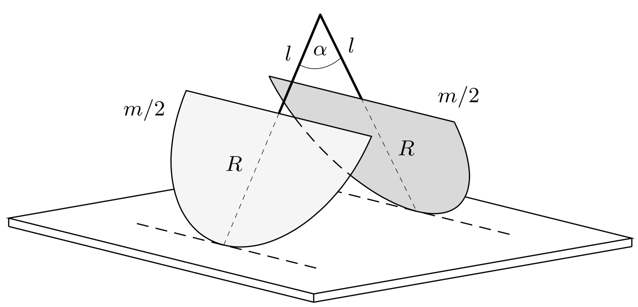
2. Let us design a simply executable measurement procedure, and let us actually perform such measurements, to determine the power exerted to keep a bicycle in motion. The bike may be driven by an electric engine or by feet. Consider all effects including air drag, friction of various components, as well as gravity. Let us try to use the simplest instruments, possibly those that can be found in a general household. As the power may depend on the speed of the bike, speed of revolution, or other parameters, let us try, instead of being general, to obtain a reproducible and precise result for a specific case. Try to determine the maximum mechanical power for the case of electric engine.

   (Gábor Veres)

3. A pointlike body is dropped on a slope of angle α on the Moon, from a given height H. The perfectly elastic body starts bouncing, along parabolic trajectories between impacts. Determine the height of the directrix of the parabolas after the n-th bounce, and find the curve connecting the focal points of the parabolas!

   (Péter Gnädig)

4. A circular, thin disk of mass m and radius R is cut into halves, according to the figure. On both pieces massless rods are fixed, along their symmetry axis, in the plane of the half-disks. The length of the rods is l. Then the free ends of the rods are attached to each other, such that the angle between them is α, and the cut sides of the half disks are parallel with each other (see figure below). This structure, placed on a flat table, may start oscillating. Calculate the frequency of the periodic motion for small displacements.



   (Géza Tichy and József Cserti)

5. J. I. *Perelman* (1882–1942) Russian astronomer in his 1929 popular science book, titled „Astronomy for entertainment" explained Lunar libration (i.e. the phenomenon that despite of its tidally locked orbiting motion, the Earth-facing side of the Moon slightly wobbles and thus not exactly the same hemisphere is seen from Earth at different times) with a reasoning similar to what follows. Let's consider a small moon with negligible mass, traveling along an elliptical trajectory characterized by its eccentricity e and the length of its semi-major axis a, around a planet with mass M. The moon rotates at a steady rate around its axis with period T_{rot}, which coincides exactly with the period T_{rev} of a revolution around the planet. Since the trajectory is elliptical, and (obeying Kepler's second law) the moon's motion is not uniform, it can be easily seen that astronomers on the planet's surface observe the moon's surface at somewhat varying angles as it travels. Interestingly, Perelman then also adds: „[the moon of this thought experiment] invariably turns one and the same face not towards the planet but towards the other focus of its orbit". Is this statement correct? Determine the magnitude of libration as seen from this other focus of the orbit, and (if not from here), find the point from where the libration looks minimal, inside the ellipse.

   (Miklós Vincze)

6. Checking on the calendar, one may observe that it is not exactly true, that the Sun rises the earliest on the day of the Summer solstice, or sets the latest on the day of the Winter solstice – but there seems to be a few days difference (actually this difference is little in Hungary). Explain the phenomenon. On which parts of Earth this difference is the largest, and why? Can a planet possibly exist, for which only one of the deviations is present, that is, the latest sunrise is not on the day of the Winter solstice, whereas it would be the earliest on the Summer solstice?

(Gergely Dálya)

7. An Earth-like exoplanet orbits around its sun on a circular trajectory. The planet evaporates, therefore it pulls a comet-like dust tail behind itself. Let us assume that the tail contains small spherical particles.
a) Let us determine the total brightness of the system as a function of time (during one revolution period).
b) Which is the orbit of the particles? Let us determine the orbital parameters.
c) Let us determine the angular speed of the dust particles relative to the planet.
d) How do the light absorption of the tail depends on the angular distance from the planet?

(Gergely Dálya)

8. A particle of mass m moves in one dimension under the influence of the force $F = -kx + a/x^3$ where k and a are positive parameters. Find the equilibrium points and analyze their stability. Calculate the frequencies of small oscillations around the equilibrium points. Show that the frequency does not depend on the energy of the particle. What is the reason of this anomalous property?

(Péter Magyar)

9. Alice and Bob are trying to calculate the shape of a uniformly loaded bridge cable in two different ways (the mass of the cable is negligible, the load is vertical with a horizontally homogeneous force density). Alice writes down the condition for local equilibrium, while Bob uses variational calculus. In his way, by fixing the cable length and summing the potential energies of the elementary load masses he concludes that the area below the cable should be minimized (isoperimetric problem). They realize with surprise that the two results are different, in particular Alice's function is a parabola while that of Bob is an arc! Reproduce their solutions. What physical settings do these results correspond to? Let's help Bob so that he could reproduce Alice's result by means of his variational method, that is, construct a potential energy functional whose minimum leads to the parabola solution.

(Géza Györgyi and Sándor Katz)

10. An airfoil (obstacle with wing-shaped cross section) is placed into a two dimensional homogeneous flow. The airfoil is obtained via Kutta-Zhukovsky transform $z + 1/z$ from a unit circle which goes through the point $x = 1$, $y = 0$. The center of the circle and the direction of the flow are free parameters. Find the position of the stagnation points (where the velocity of the flow vanishes) in terms of these parameters, provided that the velocity of the flow does not become infinite at any points (Chaplygin's condition) and no stall occurs.

(Gyula Bene)

11. When searching for the minimal surface of revolution between two rings, several different stationary shapes can be obtained. What is the physical meaning of these solutions? Consider coaxial rings of equal, unit radii with the distance d between them. Compare the area of stationary surfaces, including that of the two rings, then investigate the local stability of each solution. The eigenvalue problem can be solved, for instance, either numerically or by finding the analytic solution in the special case when there is only one smooth stationary solution and then for small deviations from it by using perturbation theory to leading order. Sketch for a few characteristic distances d the surface area ('landscape') in function space such that the stationary points are visible.

(Géza Györgyi and Sándor Katz)

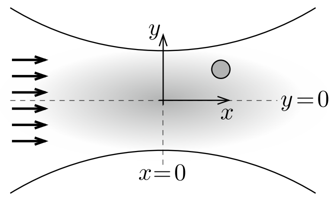
12. We are observing a 4 cm long, 2 cm diameter cylinder (wine bottle plug) using a 5-dioptre lens of 3 cm diameter. The axis of the cylinder is along the optical axis of the lens. The lens is at 30 cm distance from our eye, and the closer base of the cylinder is 2 cm behind the lens. Make a drawing of what we observe, such that the diameter of the lens is 6 cm on the drawing.

(Géza Tichy)

13. According to scientific literature in optics, there is a basic principle stating that there is no 'perfect' imaging system which magnifies or reduct. The argument is based on wave optics: the 'imaging' concept of geometrical optics is actually the constructive interference of waves which take various trajectories. But, if the distance between two points of the object is $n\lambda$, then the distance between the corresponding points of the image for N-times magnification (or reduction) is $Nn\lambda$, that is, the interference conditions drastically change. However, there is an other—also well known—concept that in case of a glass sphere with radius a a complete sphere of radius $r < a$ within the glass ball is perfectly imaged onto an other sphere of $R > a$ outside the glass sphere. Does this latter fact contradict the former principle?

(Géza Tichy)

14. A laser tweezer is a laboratory instrument, which uses highly focused laser beams to 'trap', hold or move small sized objects. The principle of the operation is that in the focal spot, the light intensity is inhomogeneous, and acts on the particle with a force that points from the low intensity region towards the high intensity region. In this problem below, the object to be trapped is a nano-ball made of latex, which is insulating, has no net electrical charge, and is much smaller than the wavelength of the light. The ball is compact, homogeneous with mass m, radius R and of relative dielectric constant of ε_r.



The nano-ball is placed into a well focused, polarized laser beam (see figure). Let us approximate the laser light in all points of the focal region as a plane wave moving in the x direction, with angular frequency of ω and with an amplitude which varies point by point. The time averaged intensity of the laser light can be approximated as

$$I(x, y, z) = I_0 \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{b^2}\right)$$

in the region of $|x| \ll a$; $|y| \ll b$; $|z| \ll b$ (here $a, b > 0$).

a) Let us determine the coordinates of the equilibrium position of the ball, that is, the point where the trapping force and the force from radiation pressure are equal. We can assume that the distance of the equilibrium position from the origin of the coordinate system is much smaller than the parameters a and b, but much larger than the ball radius R. Let us use the laws of Maxwellian electrodynamics.

b) How does the nano-ball move, if it is displaced from its equilibrium position by a small distance $d \ll b$ in the y direction? Determine the relevant parameters of the motion.

(Máté Vigh)

15. An infinite strip is constructed from resistors R, such that the resistors are arranged in a square lattice of width N (that is, there are $N + 1$ nodes and N resistors in one direction). Calculate the resistance between any two nodes of the network. Calculate exactly the resistance between two neighboring nodes which are in symmetric positions with respect to the centerline of the strip with $N = 5$.

(József Cserti)

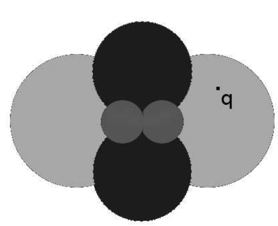
16. A long, thin, electrically charged conductive wire is placed between two grounded parallel metal plates. The wire runs parallel with the plates. The charge of the wire is Q, its length is L and the distance between the plates is D. Calculate the force acting on the wire, which is placed at d distance from one of the plates. Assume $D \ll L$.

(Péter Gnädig)

17. A table-top model of the Globe made of insulating material, is covered uniformly with finite conductivity graphite. We introduce some constant current to the sphere at Brussels, and drain the same amount of current at Budapest.
a) Determine the trajectories of the current flow in the graphite layer.
b) Find the value of the voltage between Kiev and Moscow, if the voltage between Pyongyang and Seoul is U_0.

(Péter Gnädig)

18. The figure shows a cross section of the fusion reactor of an imaginary spaceship. The point in the middle is on the boundary of all circles. (Circles of different radii are filled with three different colors on the picture.) The three dimensional structure is a solid of revolution about the vertical symmetry axis of the cross section. The walls of the circles and tori making up the reactor are ideal metals. The interior of the outer torus, painted lightgray, is hollow. Determine the potential of a static point charge in an arbitrary position inside the cavity, considering that there is no external field (the reactor itself is not operable yet...) and the walls are grounded.



(Márton Lájer)

19. The resistance of an isotropic, homogeneous, a, b, c-sided cuboid is $\rho c/(ab)$, where ρ is the resistivity (inverse of the conductivity σ), and the current flows parallel with the side of length c. In the anisotropic case, $\boldsymbol{\rho}$ and $\boldsymbol{\sigma}$ are tensors. Let us assume that the sides of the cuboid are not parallel with the principal axes of the tensors. What is the resistance of the cuboid? In which special cases, or which special geometries, can one exactly, or to a good approximation, calculate the resistance? Which combinations of the tensor components appear in the formula of the resistance? Assume that the current flows into (or out of) the cuboid through an ideally conductive metallic plate covering the full end faces of the cuboid.

(Géza Tichy)

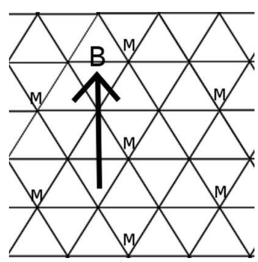
20. A copper rod is fixed firmly at the two ends horizontally, such that the length of its free section is l. The cross section of the rod is rectangular of width a and thickness b. Assume $l \gg a \gg b$. Below the rod under its longer symmetry line a small permanent magnet is placed with magnetic moment $\mathbf{m}$, such that the direction of $\mathbf{m}$ is perpendicular to the a and l-sided plane of the rod, and the distance between the center of the magnet and the rod is $d \ll l$. The variation of the magnetic field along the width of the rod is negligible. Due to the force exerted by the magnet, the rod bends slightly. How do the shape of the rod change? Let us calculate the equation determining the new, equilibrium displacement as a function of the horizontal position (in the direction of l) of the magnet. Assume that gravity is negligible, and that the displacement of the rod is much smaller than d. During the calculation you can use material constants found in tables.

(Áron Kovács)

21. A strong neodymium magnet is dropped into a vertical steel pipe. What is the velocity of the magnet during the fall? Is the uniform motion stable?

(Gyula Bene)

22. Two different types of small compasses are placed onto the points of a regular triangular lattice with unit lattice parameter. The compasses have magnetic momenta m and M, and inertial momenta θ and Θ, respectively (the picture shows the positions of the M magnetic momenta in the lattice). The compasses are free to rotate in the plane of the grid. They can be considered as ideal dipoles, which only interact with their nearest neighbors. In addition a homogeneous external magnetic field B_0 is applied in the plane of the lattice as shown in the figure. In the ground state, every compass point to the direction of the external field. Small oscillations of the compasses about their equilibrium propagate as waves in the lattice. Calculate the dispersion relation of these waves in form $f(\omega, \mathbf{k}) = 0$.



(Márton Lájer)

23. A cylindrical, perfectly heat insulating tube is divided into two equal volume regions by a heat insulating piston of mass m. The friction of the piston is not negligible; it can move parallel with the tube axis, and it is fixed at the beginning. One side of the piston, of original volume V, is filled with a monoatomic ideal gas, at pressure p and temperature T. The other side is filled with the same kind of gas, but at pressure $2p$ and temperature $2T$. Releasing the piston, it can move experiencing a constant frictional force S (that is, assume that both the velocity-independent kinetic frictional force, as well as the maximum of the static frictional force, is S). Describe the motion of the piston. What is the length of the path that it moves before stopping? Discuss the validity of the limiting case when m is very small.

(Ákos Horváth)

24. Two relativistic particles are approaching each other, their initial velocity vectors $\mathbf{V}$ and $\mathbf{v}$ are perpendicular to each other. The invariant mass of the system is ten times larger than the rest mass of the smaller object. The larger particle emits a force carrier particle, which takes some fraction of its momentum and energy. After the process, the rest mass of the original particle reduces to one seventh, and its new velocity vector will be equal to the initial velocity vector $\mathbf{v}$ of the other particle. The other, smaller particle absorbs the mediating particle, which brings it rest mass up by a factor of seven, and its new velocity vector will be equal to the initial velocity vector $\mathbf{V}$ of the original first particle. Determine the rest mass of the force carrier particle.

(Gyula Dávid)

25. Study the problem of vertical throwing in the Schwarzschild field of a spherically symmetric object. Let us find the exact differential equation between the Schwarzschild type variables r and t, corresponding to the general theory of relativity, without any approximations. How does that compare with the Newtonian formula? Let us try to solve the equation in some special cases.

(Gyula Dávid)

26. Two spaceships are falling radially towards a black hole right behind each other on a common trajectory. Their initial velocity at infinite distance is zero. At the instance of crossing the event horizon, the difference between their Schwarzschild type radial coordinates r is much smaller than the gravitational radius b of the event horizon. Let us describe the motion of one body in the local inertial system fixed to the other spaceship, and vice versa. What do the observers traveling on the spaceships experience, when their own, or the other spaceship crosses the event horizon?

(Gyula Dávid)

27. There is a spherical hollow in a Friedmann–Robertson–Walker universe with flat spatial metric and uniform matter distribution. The mass missing from the sphere is accumulated at the center of the hollow. This setting leads to an exact solution of Einstein's equations (swiss cheese model). Show that if the radius of the sphere is large enough, the following statements are true:
a) the radius of the sphere is smaller than the Schwarzschild radius,
b) if one enters the sphere, he can come out again,
c) it is not possible to reach the center of the sphere.

(Gyula Bene)

28. Consider the behaviour of a quantum particle of mass m in the 1D potential described by $V(x) = kx^2/2 + a/(2x^2)$, where k and a are positive parameters. Think of the numerous definitions, representations and calculation methods of quantum mechanics and find the one(s) best suited to attack this problem.
a) Determine the eigenfunctions and energy eigenvalues of the ground and excited states. In particular, comment on the degeneracy of the states. Interpret the results obtained. How does the complete solution related to the solutions derived for the two terms of the potential separately?
b) Can you name an existing physical system that can be adequately described by such a potential?
c) Generalize the problem to n dimensions, and discuss the special cases where $n = 1$, 2 or 3.

(Péter Magyar)

29. Let $\hat{a}^\dagger$ and $\hat{a}$ be the creation and annihilation operators of a quantum harmonic oscillator which satisfy the usual commutation relation $[\hat{a}\,,\,\hat{a}^\dagger] = \hat{I}$. Let us denote the eigenstates of the $\hat{N} = \hat{a}^\dagger\hat{a}$ quantum-number operator with $|\,n\rangle$. Let us now perform a formal Lorentz-transformation on these operators: let $\hat{b} = \hat{a}\,\mathrm{ch}\chi - \hat{a}^\dagger\,\mathrm{sh}\chi$ where χ is a fixed, real valued parameter. Prove that the operator $\hat{b}$ introduced this way and its adjoint operator can be considered as an annihilation / creation operator pair, satisfying the corresponding commutation relation. Let us determine the ground state of the operator $\hat{M} = \hat{b}^\dagger\hat{b}$, and also its first and second excited states, as a linear combination of the eigenstates $|n\rangle$ of the operator $\hat{N}$. More adept solvers can try to find the rules to present all $|m\rangle$ eigenstates, as well as the coherent states. Let us take care about the normalization of the eigenstates.

(Gyula Dávid)

30. A given state of a fermionic many-body system can be expressed in the occupation number representation with the state vectors $|n_1, \ldots, n_i, \ldots, n_N\rangle$ where $n_i \in \{0,1\}$ is the occupation number of the i-th single particle state, and N is the number of single particle states. An arbitrary operator acting on these states can be expressed as a polynomial of fermionic creation ($a_i^\dagger$) and annihilation operators (a_i). A general single particle operator acting on this Fock space can be expressed as $O_1 = \sum_{ij} O_{ij} a_i^\dagger a_j$. One can define the fractal dimension of a matrix A of dimension $M \times M$ as

$$d(A) = \frac{\ln\left(s(A)\right)}{\ln\left(M\right)},$$

where $s(A)$ is the number of nonzero elements of the matrix. (*Remark*: The fractal dimension of a generic many-particle operator is 2 since it is represented by a full matrix.) What is the fractal dimension of the densest single particle operator acting on the total Fock space and on the n-particle subspace of the Fock space ($n < N$) in the thermodynamic limit ($N \to \infty$)? How can one generalize the results to multi particle operators?

(László Oroszlány and Norbert Barankai)

31. Let us write down the solution of the *classical* field theory, defined by the Lagrangian $\mathcal{L} = \frac{1}{2}\,(\partial_\mu\,\phi)^2 - \frac{1}{2}m^2\,\phi^2$ with given initial condition $\phi_0(x)$. Based on the solution, let us also solve the model with the Lagrangian $\mathcal{L} = \frac{1}{2}\,(\partial_\mu\,\phi)^2 - \frac{1}{2}m^2\,\phi^2 + \frac{1}{3}\lambda_1\phi^3 + \frac{1}{4}\lambda_2\phi^4$ to second order of diagrammatic perturbation calculation in the coupling constants λ_1, λ_2 (that is, try to translate, if possible, the perturbation calculation to diagrams designed by yourself, and also specify the rules and values of the necessary parameters). Find the graph rules both in coordinate- and in momentum (Fourier-) space. How do the rules change, if we deal with a *quantum* field theory, instead of a *classical* one? How can we get back the classical rules from the quantum ones?

(Zoltán Laczkó)

32. It is usually said that the indistinguishability of identical particles and the Pauli exclusion principle have to be built into quantum mechanics as first principles. In the case of a system of two identical particles, we demand that the eigenfunctions of the system are simultaneous eigenfunctions of the permutation operator S, that acts on the space of wave functions $\psi(x_1, x_2)$ and performs the

$$(S\psi)(x_1, x_2) = \psi(x_2, x_1)$$

transformation. After that, the Pauli exclusion principle bans the linear combinations of the eigenstates of S corresponding to different eigenvalues of S as physically admissible systems.

We will speculate on the possibility of modification of the quantization procedure of classical mechanical systems such that the indistinguishability of identical particles are incorporated in the theory from the beginning. One possible way is the following. Let N be the number of particles of a classical system of point-like particles and its configuration space be Q^N, where Q is the one-particle configuration space. The argument stating that if the mass, charge, etc. of the particles are equal then the particles are indistinguishable is easily acceptable even classically. As a corollary, the configuration space is in fact not Q^N, but equal to the space that can be constructed by identifying those points of Q^N that can be transformed to each other by the permutation of the canonical coordinates of Q^N. In brevity, our configuration space will be Q^N/S_N, where S_N is a symmetric group, the permutation group of the series of non-equal elements of length N. Our method will be the following: instead of using the configuration space Q^N for quantization, we will use the space Q^N/S_N.

Let us turn our attention to the special case of $Q = \mathbb{R}^d$, that is when $Q^N \equiv \mathbb{R}^{dN}$. Show that the fixed points of the action of S_N form a d dimensional subspace in Q^N. What is the physical meaning of the space of fixpoints X_{CM}? Isolating X_{CM} we get that $Q^N \simeq (\mathbb{R}^{(N-1)d}/S_N) \times X_{CM}$. Let $N = 2$, then $S_2 \simeq \mathbb{Z}_2$. How $\mathbb{Z}_2$ acts on $\mathbb{R}^{(N-1)d}$? What is the nature of the topological space $\mathbb{R}^{(N-1)d}/S_N$ if we allow or do not allow the collision of particles? What are the interesting topological properties of these spaces?

Let's examine in details the simplest case when $N = 2$ and $d = 1$. Show that the configuration space is isomorphic to the half plane. The quantum mechanical description of the system in coordinate representation will be given by square integrable complex valued functions of the half plane. Nevertheless, we can not allow any such functions, because by the physical interpretability, we have to impose certain *boundary conditions* on the wave functions at the edge of the half plane. How do we have to choose these boundary conditions? Let x and z be coordinates of the half plane parallel and perpendicular to its edge, respectively. Show that if $\psi(x, z)$ is an arbitrary wave function that describes the system, then it satisfies the equation

$$\frac{\partial}{\partial z}\psi(x, 0) = \eta\psi(x, 0)$$

at the boundary for some real η. Show that the Hamiltonian of the free propagation is

$$H = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right).$$

Assume that $\psi(x, z)$ obeys the boundary condition with some fixed η. What is $\psi(x, z)$ if it satisfies the Schrödinger equation of free propagation? Discuss the η dependence of the result. At last, try to find those wave functions that may correspond to bosons and fermions.

(Norbert Barankai)

33. An investment bank, as market maker, quotes bid and offer prices for a financial instrument. The bank (correctly) evaluates the instrument at 100. The market participants (incorrectly) evaluate the same instrument, with a uniform distribution, between 90 and 110. Every market participant who evaluates the instrument above the offer price, buys a unit from the bank at the offer price. In this case the bank realizes the difference between the offer price and the correct value (100) as profit. Every market participant who evaluates the instrument below the bid price, sells a unit to the bank at the bid price. In this case the bank realizes the difference between the correct value (100) and the bid price as profit.

a) What bid and offer prices should the bank quote to maximize its profit?

Assume now that there are two market maker banks, both know about the competition, and both correctly evaluate the instrument at 100. Market participants evaluate the instrument as before. A market participant who evaluates the instrument above only one bank's offer price, buys a unit from this bank at its offer price. As before, in this case the bank realizes the difference between the offer price and the correct value (100) as profit. If a market participant evaluates the instrument above both banks' offer price, then he buys from both banks, in amounts proportional to his estimated price—value differences. The sum of the amounts is one unit. In this case each bank's profit is the product of the amount it sold, and its own offer—correct value difference. Market participant behavior and bank profit on the bid side is the mirror analogue of the offer side.

b) What bid and offer prices should the banks quote now, assuming both banks want to maximize their own profit?

(Zsolt Bihary)

34. The picture (**http://ortvay.elte.hu/2014/abrak/sandstorm.ps**) shows a sand-storm in the Mediterranean region. Using this satellite picture, estimate the size of the sand particles flying in the air.

(Zoltán Rácz)

35. Determine the Solar System's velocity of revolution around the center of the Milky Way using the public database of the Sloan Digital Sky Survey (SDSS). From the database, extract the celestial coordinates of suitably chosen stars, and their redshifts, as measured from their spectra. In the case of stars, redshifting is caused by the Doppler shift, thus it is in direct relation to line-of-sight velocity. The redshifts in the database are already corrected for the revolution of the Earth around the Sun. Individual stars follow randomly inclined orbits around the center of the Galaxy, therefore it is advisable to average line-of-sight velocities over a few square degrees, and to create a map of these averaged values. Plot this map, and, making the assumption that our Sun moves in a given direction with a given velocity in relation to this averaged 'fixed star system', determine this movement's velocity and direction in galactic coordinates. During the data analysis, use galactic (l, b) coordinates. Keep in mind that astronomical databases are not perfect. Despite all the efforts of those creating them, erroneous measurements do occur that have to be filtered out when processing.
The submitted solution should contain a detailed description of the data analysis, along with the SQL queries ran, and the source code of the program used for the processing.

Quick help for accessing the database:
The database can be accessed via the webpage http://skyserver.sdss3.org/ . Once there, click on the CasJobs link, and following a simple registration, you get complete access to the data. You can only use the SQL language to query the data. You can run queries in the Query menu of the CasJobs webpage. Before running, make sure that you are using the correct version of the database (DR7) – the version can be selected in the Context field. A detailed description of the data tables in the database can be found on the main page, in the Schema Browser menu.

The following simple query gives the Doppler redshift of a few stars:

```
SELECT TOP 100 p.l, p.b, s.z
FROM SpecObj s
INNER JOIN Star p
ON p.objID = s.bestObjID
WHERE s.SpecClass = 1
```

After running (Submit button), the results of the query are shown in the MyDB menu, from where the data are downloadable in CSV format for further processing. The data analysis may be done with a processing program written in any programming language, but the SQL language is suitable by itself for creating histograms and averaging.

The following query determines the number of stars observed by SDSS as a function of galactic latitude, with a resolution of one degree:

```
SELECT FLOOR(p.b), COUNT(*)
FROM SpecObj s
INNER JOIN Star p
ON p.objID = s.bestObjID
WHERE s.SpecClass = 1
GROUP BY FLOOR(p.b)
ORDER BY 1
```

If the COUNT(*) function is switched to AVG(s.z), we get the stars' average Doppler shift as a result, instead of their total number.
It is worth studying the 'Education', 'Schema', 'SQL Tutorial' and 'Sample SQL Queries' pages of the SkyServer portal, too.

(István Csabai and László Dobos)

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