THE 34th
– 5th INTERNATIONAL –
RUDOLF ORTVAY
PROBLEM SOLVING CONTEST IN PHYSICS
2003

The Physics Students' Association of Eötvös University, Budapest, the Roland Eötvös Physical Society and the Hungarian Association of Physics Students proudly announce the **34th** – and for the fifth time **international** –

Rudolf Ortvay Problem Solving Contest in Physics,
from 31 October 2003, through 10 November 2003.

Every university student from any country can participate in the Ortvay Contest. PhD students compete in a separate category. The contest is for individuals: solutions sent by groups of students are not accepted. The name, the university, the major, and the university year should be indicated on the solutions. Pseudonyms and passwords cannot be used: each contestant has to use his/her own name. The problems can be **downloaded** from the webpages of the Ortvay Contest

http://ortvay.elte.hu/

http://www.saas.hu/ortvay

in Hungarian and English languages, in html, LaTeX and Postscript formats, from **12 o'clock (Central European Time, 11:00 GMT), Friday, 31 October 2003**. The problems will also be distributed by local organizers at many universities outside of Hungary.
Despite all the efforts of the organizers, it may happen that some unclear points or misprintz stay in the text. Therefore it is very useful to visit the webpage of the contest from time to time, as the corrections and/or modifications will appear there.
Each contestant can send solutions for up to 10 problems. For the solution of each problem 100 points can be given. *Each problem should be presented on (a) separate A4, or letter-sized sheet(s). The contestants are kindly asked to use only one side of each sheet. Solutions written by pencil or written on thin copy-paper will not be accepted as these cannot be faxed to the referees.*
Any kind of reference material may be consulted; textbooks and articles of journals can be cited. Computer programs appended to the solutions should be accompanied by detailed descriptions (what computer language it has been written in, how to use the program, which parameters can be set, what notations are used, how to interpret the output figures and graphs, etc.) They can be enclosed on floppy disks, or sent via email to the addresses below. Solutions can be sent by mail, fax, or email (in LaTeX, TeX or Postscript formats—or, if they contain no formulae, in normal electronic mail). Contestants are asked not to use very special LaTeX style files unless included in the sent file(s).

Postal Address:

Fizikus Diákkör, Dávid Gyula,
ELTE TTK Atomfizika Tanszék,
H-1117 Budapest, Pázmány Péter sétány 1/A
Fax: Dávid Gyula, 36-1-3722775 *or* Cserti József, 36-1-3722866
E-mail: dgy@ludens.elte.hu *or* ortvay@saas.city.tvnet.hu

Deadline for sending the solutions: 12 o'clock CET (11:00 GMT), 10 November 2003.

Contestants are asked to fill in the form available on our webpage after posting their solutions. It will be used for identification of contestants and their solutions. **Without filling in the form, the organizers cannot accept the solutions! The form is available only on 10th and 11th November.**

The contest will be evaluated separately for each university year, according to the total number of points. The referees reserve the right to withhold, to multiply or to share some prizes. Beyond the money prizes given for the first, second, and third places, honorable mentions and special prizes for the outstanding solutions of individual problems can be awarded. This is why it is worthwhile sending even one or two solutions.
The announcement of the results will take place on 5 December, 2003. The detailed results will be available on the webpage of the contest thereafter. Certificates and money prizes will be sent by mail. We plan to publish the assigned problems and their solutions in English language—to which the contribution of the most successful participants is kindly asked. The volume is planned to be distributed all over the world with the help of the International Association of Physics Students, as well as the contestants themselves. We hope this will help in making the contest even more international. Wishing a successful contest to all our participants,

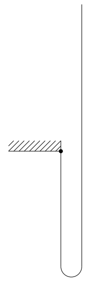
the Organizing Committee:

Gyula Dávid, Attila Piróth, József Cserti
(Eötvös University, Budapest, Hungary)

1. Time passes at a snail's pace these days — rainy and misty early-November may make you feel. Neglect the evaporation of the oceans, and assume that it is raining all over the world. How is the length of the day affected?

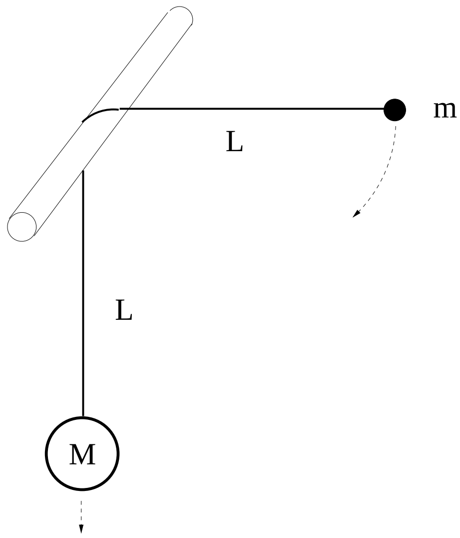
(Gábor Veres)

2. A rope fixed at one end is released from its initial position shown in the figure. (The shape of rope can be approximated by two straight vertical lines connected by a small semicircle.)
a) Describe the motion of the rope up to the point where its free end gets to the lowest position.
b) Is mechanical energy conserved in the process? If not, where is energy dissipated and why?
c) How does the radius of the circle connecting the vertical parts of the rope change with time (in a first approximation)?



(Dezső Varga)

3. A thin rod (of negligible radius) is fixed horizontally. Two mass points, m and M are attached to the two ends of a cord of length $2L$, and then the cord is placed on the rod in such a way that the body of mass M is hanging vertically on a length L of the cord, while the other one (of mass m) is at rest in the same height as the rod, at a distance L from it. When the bodies are released, the cord starts to slide on the rod, and then sticks to it. Assume that the rod and the cord are made of special material such that the coefficient of dynamic friction between the two is negligible, while that of static friction is very large. Furthermore, assume that the body of mass m does not get entangled with the vertical cord holding the other body.
What mass ratio M/m is necessary if the cord is to stay taut throughout the motion?



(Péter Gnädig, based on a problem of the International Physics Olympiad)

4. Find the equilibrium position of a particle moving in the one-dimensional potential $U(x) = ax^{-12} - bx^{-6}$ $(a, b > 0)$, and also the frequency of small-amplitude oscillations about this point. How is this frequency modified if the amplitude of the displacement is increased (though still small)? What motions are expected in two dimensions for very small but not too small displacements, if the potential is given by $U(r) = ar^{-12} - br^{-6}$?

(Péter Pollner)

5. Several space projects are based on the idea of placing a spatial probe into one of the Lagrangian points of the Earth-Moon system. The Lagrangian points are on the orbit of the Moon (assumed to be circular), making an equilateral traingle with the two astronomical bodies. Show that such points are points of stable equilibrium for the probe. Determine the frequency of the small-amplitude normal mode oscillations of the probe about the Lagrangian point. What trajectory does it trace out?

(Gyula Dávid)

6. Two bodies of mass m are placed on an inclined plane, a cylindrical tube of outer radius r and relatively thin wall, and a solid cylinder of radius r (made of low-density material). The two bodies touch each other and the inclined plane. Initially both are at rest, and the solid cylinder is placed upper than the hollow one. The coefficient of friction is $\mu = 1/2$ for each pair of surfaces.

For what angles of inclination will the two bodies roll without slipping?

(Péter Balogh)

7. We have two pieces of unstretchable ropes of length S, negligible mass, and circular cross section of radius R. The ropes are attached to the ceiling; their points of suspension are a distance D apart. To the other ends of the ropes is attached a thin rod of uniform mass distribution. The system thus obtained is similar to the acrobats' trapeze. Starting from the equilibrium position of the system, the rod is turned N times around its vertical symmetry axis, and so the ropes get wound around one another. Assume that no torsion energy is stored in the ropes, and that the resistance of the media can be neglected. How long does it take for the system to get unwound (in the absence of any external action)? Describe the motion if friction between the ropes is considerable. (To obtain explicit numerical results, use the following data: $S = 3$ m, $D = 1$ m, $L = 2$ m, $R = 1$ cm, $N = 20$; and the coefficient of friction $\mu = 0{,}5$.)

(Merse Előd Gáspár)

8. Astronomers discover two comets of equal mass approaching the sun on parabolic orbits. They find that the angular momentum vectors of the two comets are equal in magnitude but opposite in direction.

   The two comets collide in their perihelia, and fall into several pieces that start to move in different directions with velocities of equal magnitude. Determine the envelope surface of the debris as it moves on.

(Based on a problem by Prof. Mihail Sandu)

9. What would the gravitational field of a sphere of uniform mass distribution be like (both inside and outside the sphere), if the exponent in Newton's law were $n \neq 2$?

   Imagine that the Creator has already written the Book of mathematics (referred to so many times by Paul Erdős), created the Sky, the Earth and also the asteroids — but has not yet decided the explicit form of the law of gravitation.

   How should (s)he choose the exponent n in the law of gravitation if (s)he wants to assure that the acceleration due to gravity at a height h above and below the surface of a purely golden asteroid of radius R and uniform mass distribution be equal if (and only if) the value of h/R is the famous Golden section (which, needles to say, is in the Book)?

(Péter Gnädig)

10. Let's construct a tower of height 300 metres ($\sim$ 1000 feet) with the simple guiding principle that it should resist the toppling power of wind. (For simplicity, assume that the horizontal cross section of the tower is square; the edges of the squares are parallel, and their centres are on the same vertical line. The wind is supposed to blow along one pair of edges of the squares.) The shape of the tower should be such that at any given height the torque from the wind pressure on the portion above be compensated by the torque from the weight of the portion above. What function describes the shape of the tower? Assume that the wind pressure at the base of the tower is 1500 $\mathrm{N/m^2}$, and increases linearly to 4500 $\mathrm{N/m^2}$ at the top. What is the width of the 300-metre tower at its base?

(Zsolt Frei)

11. The circular hole in the middle of a drum (considered as a membrane) is darned with an elastic material, different from that of the drum. How are the eigenfrequencies modified?

(József Cserti)

12. Give some methods to estimate the lifetime of the processor fan of a computer. (Fans usually stop or break down because their axle gets worn down due to rapid rotation and lack of lubrication.)

(Péter Pollner)

13. For a given system of interacting particles the potential is the nth order homogeneous function of the position vectors. An example is a system of particles of different charges interacting via electrostatic forces. Show that the total energy is zero in equilibrium. Find some other examples of systems for which the above theorem is valid — and also some for which it is not.

(Géza Tichy and Péter Gnädig, based on a secondary-school problem)

14. Certain liquids can be layered one above the other with a sharp interface between them. For liquids of different surface tension an interesting phenomenon can be observed. Blow bubbles of different size into the lower liquid and observe their behaviour near the interface. Investigate and explain the phenomenon.

(Zsuzsa Rajkovits)

15. Give a quantitative estimate on the magnitude of tidal phenomena (spring tide, neap tide), and try to explain the well-known extreme values (Saint-Malo Bay, the estuary of the river Saint Lawrence, where tidal fluctuations can reach 10 to 15 metres).

(András Pál)

16. The atmosphere of a planet is of constant temperature, and $c_P/c_V = 1$, therefore the pressure profile is of the form $\exp(-h/h_0)$ as a function of the altitude. The atmosphere is thin, in the sense that the radius of the planet is much larger than h_0.

a) Determine the velocity of long wavelength ($\lambda \gg h_0$) gravity waves propagating in the atmosphere. What are the eigenfrequencies of the atmosphere's oscillations?

b) Determine the eigenfrequencies of the radial (spherically symmetric) oscillations of the atmosphere. What is the dispersion relation for the waves propagating vertically upwards? What happens to the energy of the upward-going waves, how is it reflected or dissipated? (Certainly, sound waves cannot propagate in the outer space, being void of air).

(Dezső Varga)

17. A small body mass m, spherically symmetric mass distribution, moment of inertia Θ, and electric dipole moment $\mathbf{d}$ is moving in a uniform magnetic field $\mathbf{B}$. Only electromagnetic forces act on it.

a) Find some constants of motion.

b) Describe the motion of the body if initially its velocity is zero, and its angular velocity ($\boldsymbol{\omega}_0$) is parallel to $\mathbf{B}$ and perpendicular to $\mathbf{d}$.

(Péter Gnädig, based on a problem of the Eötvös contest)

18. Consider our world as a two-dimensional spherical surface where electrostatics is described by the spherical Poisson equation.

a) Determine the potential of the electrostatic field due to a point charge. Is there any problem with the solution? Is it possible that only one point charge should exist in the world? What is the total charge of the world?

b) Define point-like multipoles of order ℓ and determine their potential.

c) Solve the following Dirichlet problem. The potential is given by ϕ_1 and ϕ_2 on two circles of the sphere. Find the electrostatic potential in the three regions of the world in terms of the Fourier coefficients of ϕ_1 and ϕ_2, if charges can be present only on the circles. What is the linear charge density of the circles?

(Viktor Eisler and Szilárd Farkas)

19. Determine the induced magnetic moment of a conducting sphere of radius R when set into rotation with angular velocity ω. (Within the conductor, electrons of charge $-e$ and mass m are free to move.)

Estimate the ratio of the earth's magnetic field that can be the result of this mechanism.

(Péter Gnädig)

20. Ampère, in his "*Memoir on the Mathematical Theory of Electromagnetic Phenomena Uniquely Deduced from Experience*", published in 1827, recounts the following experiment. *Let us consider three wires of circular form, each in the same plane. Let the centres of the circles, denoted by O', O'', and O''', lie on the same straight line and denote by R', R'', R''', respectively, their radii. Let us then make circulate the same current, oriented in the clockwise direction, in the three circuits. Then the total force acting on the conductor in the middle vanishes, provided the geometrical data satisfy the condition*

$$\frac{R'}{R''} = \frac{R''}{R'''} = \frac{\overline{O'O''}}{\overline{O''O'''}}.$$

The same remains true if the direction of the current in the middle conductor is reversed.

Verify that Ampère's observation is consistent with the laws of electromagnetism.

(Péter Horváthy)

21. The edges of an infinite square lattice are identical resistances R of the same length L. The network of resistors is then cut along two parallel diagonals a distance $N\sqrt{2}L$ apart (where N is an integer). The obtained infinite strip (of width $N\sqrt{2}L$) is wrapped up into a cylinder with the short (finite) side as circumference, ie the points hitherto opposite are shorted. Determine the resultant resistance between any two points of the new network — or at least show that an exact calculation can be performed. Try to give explicit numerical result for some pairs of points, if $N = 2$ and $N = 3$.
Suggested reference: Monwhea Jeng, *Random walks and effective resistances on torodial and cylindrical grids*, Am. J. Phys. **68**, 37-40 (2000).

(Merse Előd Gáspár)

22. Show directly that the electromagnetic fields of a point-like dipole with harmonic time dependence (see e.g. Jackson: Classical Electrodynamics, 1975; 1998) obey the Maxwell equations in the *whole space*.

(Mihály Benedict)

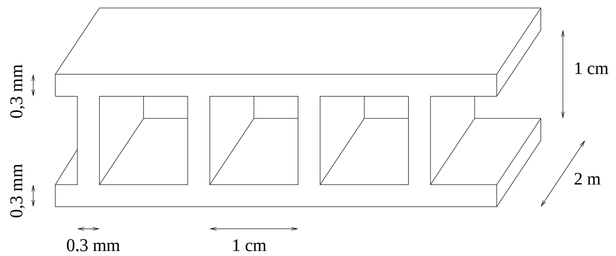
23. The shape of the rising/setting Sun is known not to be circular but somewhat oblate. Estimate the degree of oblateness of the solar disk. What atmospheric conditions are necessary to have a non-connected image of the sun? (This phenomenon is rather rare.)

(András Pál)

24. Design a passive optical device to focus the most possible sunlight on a given surface between sunrise and sunset. A practical motivation to this problem is that photovoltaic cells are still expensive, therefore all cheap possibilities to increase their performance should be exploited.

(Zénó Farkas)

25. Our terrace has been covered with a *lexan* plate (also used to cover bus stops). This consists of two transparent sheets of width 0.3 mm, separated by a distance of 1 cm. The perpendicular cell walls (as spacers) running between the two sheets are of the same material and the same width, and are spaced 1 cm apart. The spacers run along the direction of the inclination of the roof. How is the moon seen through the roof, viewed at different angles?



(István Csabai)

26. Blow soap bubbles, and place them on a liquid surface and a plain plate. When illuminated by sunlight, bright spots can be observed on the bubble. Investigate and explain the phenomenon.

(Zsuzsa Rajkovits)

27. In Arthur C. Clarke's novel *2010* (and the film based on it) super-intelligent (and super-high-tech) aliens transform Jupiter into a star so as to provide ideal environment for the development of the creatures living in the ice-covered oceans of the Jovian moon Europe. The last images of the film touchingly show two suns shining on the terrestrial cities.

Forget about the technical difficulties of transforming a planet into a star (and also about the fact that according to our present-day understanding of stellar physics, the mass of Jupiter is too small for this transformation — the aliens must known stellar physics and technology much better than us), and examine its effects on the solar system. Assume that the aliens are economic, ie the luminosity of the new star is just enough to melt the ice of Europe and warm the ocean to the stable temperature of terrestrial oceans.

How bright is the new star (called Lucifer) as viewed from the earth? Is it visible on the day sky? How much is the terrestrial climate affected? What effects does this change have on the different objects in the solar system?

(Gyula Dávid)

28. **To the memory of György Marx**

In Nordstörm's Lorentz covariant theory of gravitation (1912), the gravitational force on a point-like particle of rest mass M is given by

$$F_k = M\,\partial_k \Phi,$$

where $\Phi(\mathbf{r}, t)$ is a 4-scalar field.

a) Show that the particle's 4-acceleration given by the above force equation is indeed independent of the mass.

b) Consider a static Nordstörm field (ie one that is time-independent in a given inertial frame). Calculate the particle's 3-acceleration, and show that it is independent of its rest mass.

c) Examine the problem of horizontal projection in a uniform, static Nordstörm field $\Phi = gz$. The projectile is launched at height h with initial velocity v_0 (in the horizontal direction). Calculate its trajectory and the time of the fall. Determine the paths traced out in time t by projectiles set in motion with different initial speeds. What is the particle's velocity for $t \to \infty$? Under what assumptions are the results of classical Newtonian mechanics recovered? What is the order of discrepancy between the two theories if g is the same as the acceleration due to gravity on the earth's surface, the initial altitude is some ten metres and the initial velocity is of the order of a few times ten m/s?

d) Repeat the calculation for reciprocal mass force given by

$$F_k = \frac{1}{M}\,\partial_k V(\mathbf{r},t),$$

where V is another 4-scalar field. Determine the velocity at $t \to \infty$ of a body projected horizontally in a uniform force field. Don't you find the result surprising?

(Gyula Dávid)

29. One signal type that present-day gravitational wave detectors might be able to detect is gravitational bremsstrahlung, created when two compact stars on hyperbolic paths pass by each other. The characteristic frequency of the emitted gravitational radiation is $f = v/b$, where b is the distance of closest approach, and v is the relative speed in that point.

Estimate the rate of gravitational bremsstrahlung signals produced by a typical globular cluster within the frequency range of the terrestrial gravitational wave detectors (between 100 and 3000 Hz). For simplicity, assume that the cluster contains $N = 10^6$ stars, distributed uniformly over a sphere 20 ly across. First study the "perfect gas" approximation, and then take into account deflections due to gravitation.

(Bence Kocsis and Merse Előd Gáspár)

30. In an early tentative version of general relativity, Einstein and others tried to describe the gravitational effects on the structure of the space-time of special relativity by assuming that the speed of light was not a universal constant, just a quantity varying over space as determined by the local gravitational potential. This lead them to write the action integral of a free particle (ie one under the influence of gravitation only) as

$$S = -mc_0 \int ds = \int (-mc_0)\sqrt{c(\mathbf{r},t)^2 - v^2}\,dt,$$

where v is the particle velocity, c_0 is the speed of light in gravitation-free space, and $c(\mathbf{r},t)$ is the local speed of light (showing spatial variation).

a) Consider the integrand in the last expression as a Lagrangian of classical mechanics, and derive the equations of motion.

b) Derive the equations of motion in covariant formulation. (Attention!)

c) What is the field equation for $c(\mathbf{r},t)$ if the principle of equivalence is to be satisfied?

d) Examine the special case of static gravitation $[c(\mathbf{r},t) = c(\mathbf{r})]$. Derive the particle's equation of motion, and find the energy integral. Formulate the optical equivalent of the problem (using Fermat's principle).

e) Show that this theory can be considered as the description of geodesic motion in a Riemannian space of special (curved) metric. Calculate the energy-momentum tensor of the continuous distribution of matter, ie the source of the field. Try to find the Lagrangian density.

(Gyula Dávid)

31. It is well known that rigid bodies (in a strict sense) do not exist within the frameworks of relativity and quantum theory. It is also widely known that the classical mechanics of systems of particles can be extended to Riemannian manifolds, however, rigid bodies can exist only on homogeneous manifolds. As opposed to a strictly rigid body, a "loosely rigid" one (ie a system of mass points connected by strong springs) can move freely on any Riemannian manifold, and in this case a new effect (compared to the motion of free particles) can be observed: the non-homogeneous manifold exerts a force on the "loosely rigid" body at rest. What does this force depend on? How many "loosely rigid" bodies are at least necessary to produce it? Find a model in which the distance-dependence of the force is the same as in Newton's law of gravitation. Assume that on the scale of the rigid body the curvature of the manifold changes only slightly.

(Balázs Pozsgai)

32. A sphere made of diffusive material contains radium in a uniform distribution. The radium decays, and a part of the created radon can escape into the air. Calculate the radon activity of the sphere as a function of its radius, if the diffusion constants and the radium concentration (activity/volume) of the sphere are known. (Use parameters of realistic order.)

(Ákos Horváth)

33. A ^{8}Li nucleus of kinetic energy 100 MeV is about to make a head-on collision with a fixed heavy target, a positively charged $Z = 82$ nucleus. The ^{8}Li nucleus suddenly takes up 2333 keV of energy from the electromagnetic field of the nucleus. A part, 2033 keV is used to liberate a neutron — thus the ^{8}Li is transformed into ^{7}Li plus a neutron. The remaining energy of 300 keV increases the kinetic energy of the parts. Describe the trajectory of the center of mass. (Energy absorption is, of course, accompanied by momentum transfer, but this can be neglected in the present problem.)

(Ákos Horváth)

34. **To the memory of Edward Teller**

Consider a two-dimensional, cylindrically symmetric harmonic potential well, $V(x, y) = k(x^2 + y^2)/2$, in which electrons move. (The electrons do not interact with one another.) The potential well is deformed by the normal oscillations (canonical coordinates: Z and P) of the ions creating the well, leading to

$$V(x, y) = \frac{k + \alpha Z}{2} x^2 + \frac{k - \alpha Z}{2} y^2.$$

a) Write down classically the equations of motion for the coupled system of an electron and a phonon, and solve it in adiabatic approximation (the phonon moves slowly compared to the electrons). Interpret. What condition must α staisfy so that the "molecule" (the system of the phonon and the electrons) be stable?

b) Solve the one-electron problem quantum mechanically, up to first order in α. (What is the small parameter?) At what occupation level is the Jahn-Teller effect observed?

c) Restrict the Hilbert space: consider only the two lowest electron levels; this way one can go beyond first order in α. When do we find dynamic and when static Jahn-Teller effect?

Suggested reference: J.C. Slonczewski: *Theory of the Dynamical Jahn-Teller Effect*, Physical Review **131**, 1596-1610 (1963).

(Titusz Fehér)

35. A circular ring of radius R and negligible cross-section is placed into a uniform external field $\mathbf{B}$. The nonrelativistic Hamiltonian of the electron moving within the ring is of the form

$$H = \frac{1}{2m} (\mathbf{p} - e\mathbf{A})^2 - \mu \mathbf{B} \boldsymbol{\sigma},$$

where the first term is the kinetic energy, the second one describes the Zeeman interaction, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is a vector formed from the Pauli spin matrices, and $\mathbf{A}$ is the vector potential. The external field $\mathbf{B}$ is not necessarily perpendicular to the plane of the ring.

Determine the energy levels of the electron.

(József Cserti)

36. A two-state quantum system (qbit) travelling at a uniform velocity of v ($\ll c$) (along a rectilinear path) is studied in a two-dimensional space-time of quantum nature. The initial state of the qbit is $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, where $|0\rangle$ and $|1\rangle$ are energy eigenstates. The metric of the space-time is described by the vector $\mathbf{a} = (a_1, a_2, a_3, a_4) \in \mathbb{R}^4$ as

$$g_{\mathbf{a}} = \begin{pmatrix} a_1 + 1 & a_2 \\ a_3 & a_4 + 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a_1 + 1 & a_3 \\ a_2 & a_4 + 1 \end{pmatrix}.$$

In state $|g_{\mathbf{a}}\rangle$ the metric is $g_{\mathbf{a}}$ with a probability of 1. The space-time fluctuations are described (in a simplistic way) by the density matrix $\rho_g = \int d\mathbf{a} f(\mathbf{a}) |g_{\mathbf{a}}\rangle\langle g_{\mathbf{a}}|$, where $f(\mathbf{a}) = \exp(-\mathbf{a}A\mathbf{a})$, and A is proportional to the unit matrix. The constant of proportionality assures that the fluctuations be small. In state $|g_{\mathbf{a}}\rangle$ the time evolution of the energy eigenstate $|k\rangle$ of the qbit is given by

$$|g_{\mathbf{a}}\rangle|k\rangle \to |g_{\mathbf{a}}\rangle|k\rangle \exp(i\omega_k \tau_{\mathbf{a}}),$$

where ω_k is the frequency of the kth energy eigenstate, and $\tau_{\mathbf{a}}$ is the proper time in the metric described by $\mathbf{a}$. Show that the off-diagonal elements of the reduced density matrix of the qbit fall off exponentially with time. (*Hint:* Expand $\tau_{\mathbf{a}}$ in terms of $\mathbf{a}$ up to second order.)

(Győző Egri)

37. Alice wants to send a secret message to Bob, letting him know at which of the four places: I, X, Z and Y she would meet him. She wants to be sure that Eve, who is very eager to know the place, should not be able to learn the secret. Luckily, there is a source producing a pair of spin-half particles A and B, so that A goes to Alice, and B to Bob. The collective spin state of the pair is, however, a singlet state with total spin 0, that is $|\Psi\rangle = (|+\rangle_A |-\rangle_B - |-\rangle_A |+\rangle_B)/\sqrt{2}$. Alice can choose to perform one of the following four transformations on her particle: I: no transformation; X: flipping the spin state (that is $|+\rangle_A \rightleftarrows |-\rangle_A$); Z: changing the relative phase of the states $|+\rangle_A$ and $|-\rangle_A$ by π; or performing $Y = XZ$, the product of the last two. Once this is done, she sends particle A to Bob, who then flips the state of *his* particle, B, if the spin just received from Alice is in state $|-\rangle_A$, and he does nothing, if it is in $|+\rangle_A$, while he keeps the state of A unchanged. Then Bob performs two *measurements*: one on particle B, then another on particle A.

    Which quantities (observables) should he measure, to get Alice's message with certitude? Explain why Eve cannot decipher the message, even if she intercepts the particle sent from Alice to Bob. Why is it possible, in principle, to perform all the transformations before Bob's two last measurements, without introducing any other changes in the states?

    (Mihály Benedict)

38. Estimate the light intensity necessary to create the light sabres of Star Wars. In other words, when does a beam of light behave as a rod of finite length? Examine the usability of such an arm, eg the force necessary to hold it in hand etc.

    (Gábor Veres)

39. Buying PC farms is getting more and more popular these days. Lettice Lettuce would like to buy one, to have her programs run parallelly. She would like to buy an even (n) number of computers and connect them in a ring topology (in which each computer can communicate with its two neighbours). The state of each machine is described by a number showing the number of operations already performed. The states of computers of even (odd) index can take on non-negative integer (half-integer) values. The initial state is 0, $\frac{1}{2}$, 0, $\frac{1}{2}$, ... Computation is performed in discrete steps; with probability p the machine performs the step in one unit of time, and with probability $1 - p$, in k units of time. A machine can carry on its computations only if its neighbours are both ahead of it, otherwise it stays idle. The performance of the farm is defined as the average number of computers working simultaneously. Help Lettice to predict the performance of the farm.

    a) Lettice can afford to buy 4 computers. Calculate the performance as a function of p, if $k = 2$.

    b) Lettice expands the farm. What performance will she get for $n = 6, 8, \ldots$, if $k = 2$?

    c) Find the performance for arbitrary values of k and n.

    Note: Lettice is happy with exact as well as approximate results. However, she considers the results of simulations as mere illustrations.

    (Győző Egri)

40. If the windows are closed, and several people are present in an average (dormitory) room for a long time, it starts to smell foul. If, moreover, the sun is beaming into the room across the window, the air gets warm — so the room will slowly get hot and stinky. If, on the other hand, the window is let open for a good while, the air gets cold.

    Work out a compromise strategy (ie a schedule for the time and duration of airing the room) to make the room the least possible unbearable. Take into account the following, generally realistic facts: people are usually not in the room during daytime, only in the afternoon/evening; the temperature of the outer environment changes according to the empirical findings, a greenhouse effect sets in (literally) etc.

    Heat conduction and mixing of gases are known to be best described by the diffusion equation, if air is not moving. If, however, air currents are present, they stir the air in such a way that both temperature and gas concentrations become more evenly distributed.

    How should the strategy be changed if a university lecture hall is to be made more bearable? The above described effects are also present here, however, crowds invade the hall at other hours of the day.

    (András Pál)

41. The inhabitants of the far-away planet Quumbrantapaguia set a discovery spacecraft on orbit around the earth to search for intelligent life forms on our planet. The material and communicational bases of their civilisation are completely different from ours; fire, electricity, radio, speech, art or science has no significance for them. Their only detector is capable of following the trajectories of living creatures — however, speed and acceleration has no meaning for the Quumbrantapaguians. Moreover, the device detects only motions requiring biological activity. Are there forms of motions that — after the pattern analysis of the trajectories — give undisputed evidence for the intelligence of the human race? Do they find other intelligent races on our planet?

    (Imre Jánosi)

42. Near the shores of the island Focus, enveloped in concentric atolls, famous for its myriads of cocopalms, on the bottom of the crystal clear sea is lying Holy Snell, the three-master boat of the feared pirate captain, Joe the Halfhead, ever since its legendary shipwreck. J. B. Snoop of the University of Fahrehwey, the well-known discoverer of the Atlantis observatory (see Problem 25 of 1991's Ortvay Contest), and his assistant, Fin Geryn Ev Reepie would like to discover the Captain's treasure. Their airplane is gliding at a constant height, along a straight line, above the sea. With bulging eyes, the two treasure-hunters are scanning the bottom of the sea running the in their face under the mirror flat surface. Suddenly they spot out the sunk boat. After the first excitement Fin Geryn Ev Reepie turns pale and screams: "My goodness! It has moved!" And indeed: in front of the eyes of the astonished treasure-hunters, the pirates' boat starts to move in a weird way...

    Or at least this is what we read in the slightly crumpled, therefore hardly readable e-mail that arrived from the University of Fahrehwey. That is why the treasure was not brought to the surface, and this is the only reason why the organisers cannot offer Joe the Halfhead's most beautiful gold-rimmed contact lenses to the winners of the Ortvay contest.

    Only one question remains: on what trajectory did the researchers of the University of Fahrehwey see the boat, Holy Snell move?

    (József Cserti, Gyula Dávid, Attila Piróth)

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